On a Routing Problem within Probabilistic Graphs
and its application to Intermittently Connected Networks

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***Abstract*— Given a probabilistic graph G representing an intermittently connected network and routing algorithm A, we wish to determine a delivery subgraph $G[A]$ of G with at most k edges, such that the probability $Conn_2(G[A])$ that there is a path from source s to destination t (in a graph H chosen randomly from the probability space defined by $G[A]$) is maximized. To the best of our knowledge, this problem and its complexity has not been addressed in the literature. Also, there is the corresponding distributed version of the problem where the delivery subgraph $G[A]$ is to be constructed distributively, yielding a routing protocol.**

Our proposed solution to this routing problem is multi-fold: First, we prove the hardness of our optimization problem of finding a delivery subgraph that maximizes the delivery probability and discuss the hardness of computing the objective function $Conn_2(G[A])$; Second, we present an algorithm to approximate $Conn_2(G[A])$ and compare it with an optimal algorithm; Third, we focus on intermittently connected networks, and model the users' mobility within them; and Fourth, we propose an edge-constrained routing protocol (EC-SOLAR-KSP) based on the insights obtained from the first step and the contact probabilities computed in the third step. We then highlight the protocol's novelty and effectiveness by comparing it with a probabilistic routing protocol, and an epidemic routing protocol proposed in literature.

I. Introduction

The mobility of users forming a mobile wireless network causes changes in the network connectivity and may even lead to intermittently connected (or, temporally disconnected) networks. On one hand, nodal mobility may increase the overall network capacity [1]. On the other hand, it may make it challenging to locate users and route messages within the network. Although many proactive, reactive, and hybrid approaches have been suggested in the literature for various types of mobile wireless networks, mobility of nodes is still considered a big threat to final protocol performance.

While the authors in [2], [3] proposed data infusion techniques suitable for intermittent connectivity, another direction of research has been the better understanding of the underlying user mobility itself, that may be leveraged upon in routing decisions and location predictions. Researchers have analyzed mobility traces to various degrees and have suggested numerous practical mobility models. The insights thus gained has also been instrumental in proposing several probabilistic routing protocols [4]–[6]. At the same time, profiling wireless users based on their mobility has also been proven beneficial to routing. The suggested approaches to profiling [7], [8] have however varied, as well as the details of the mobility information required.

In our previous work [9] we had established a mobility profiling framework based on the sociological influences on wireless users' movements and presented a technique for processing sporadic mobility trace data more generally found amongst real traces. We validated our sociological claims via an empirical study of a year long mobility trace data obtained from ETH Zurich campus. In particular, we observed that most wireless network terrains (campuses) comprise of a list of sociologically significant places (cafeteria, library, dormitories) that we refer to as "hubs", and most users move amongst a few selected hubs over an extended period in time that we refer to as the user's sociological "orbit". We applied a Mixture of Bernoulli's distribution using the Expectation-Maximization algorithm to generate "mobility profiles" for individual wireless users involving a probabilistic movement among hubs, and highlighted the usefulness of our mobility profiles in performing location predictions with higher accuracy. We further demonstrated its use in probabilistic routing within Intermittently Connected Mobile Ad hoc Networks (ICMAN) by proposing and analyzing a suite of Sociological Orbit aware Location Approximation and Routing (SOLAR) protocols in [10], [11].

In this work, we aim to provide in detail a mathematical model for computing the contact probabilities and subsequently the delivery probabilities that were used by our SOLAR protocols, based on our mobility profile information and the hub transitional probabilities. To facilitate our discussion, we shall define a couple of terms:

1) *contact probability*: The probability of two nodes ever coming within each other's radio range (in contact) during the entire observed time window.
2) *delivery probability*: The probability of a source delivering a packet to a destination via all possible paths of intermediary nodes that come "in contact" with their predecessors and successors in their respective paths.

The rest of this paper is outlined as follows. We start off by formulating our routing problem in Section II and then discuss its complexity in Section III. We then present an approximation algorithm for computing the delivery probability in Section IV and compare its performance with an optimal solution. In Sections V we present a mathematical model for computing the pairwise user *contact probability*, and propose and analyze an edge-constrained routing protocol called EC-SOLAR-KSP in Section VI which makes use of such contact probabilities. In Section VII we discuss other related work and finally conclude this paper in Section VIII.

II. A Routing Problem on Random Graphs

In many different wireless network scenarios and applications (DTN, MANET, ICMAN, ...) the minimal piece of information that a network node can gather locally is the probability that it can deliver a packet to another node in the network. This so-called "contact probability" can also be estimated/predicted with a good mobility model, depending on the specific application we are working on [7], [8]. Some prior studies have used node contact probabilities to devise routing protocols [5], [11].

In this section, we will rigorously formulate this problem. A good solution to this problem can be used to devise provably (near) optimal solution to the probabilistic routing problem. Moreover, it can also be used as a benchmark to compare probabilistic routing protocols in the literature.

Define a directed graph $G = (V, E)$ whose vertices represent nodes in the network under consideration. For each pair of nodes u and v, let $p_{(u,v)}$ denote the probability of u being able to deliver a data packet to v given some practical constraint(s) and/or some mobility model for nodes' movements in the network. For simplicity, we assume that all these contact probabilities are independent. If not, the problem becomes too complex to be useful. This assumption is not too restrictive, as we will demonstrate with our routing protocol in a later section. (Note that, in this section we are not yet concerned about the question of how to compute $p_{(u,v)}$. The problem of computing the contact probabilities is orthogonal to the routing problem on random graphs that we are formulating. In a later section, we will present a model for estimating these probabilities.)

The edges of G are precisely those pairs $e = (u, v)$ for which $p_e > 0$. Consider a source s and a destination t in G, and the problem of finding the best way to deliver a packet from s to t in the network. The obvious objective is to maximize the delivery probability of the packet. A broadcast (epidemic-like) routing algorithm seems to be best in terms of maximizing the delivery probability; however, broadcasting imposes a high cost in terms of data and processing overhead. It is thus natural to formulate a problem investigating the tradeoff between delivery probability and overhead.

Consider any routing algorithm/protocol A run by all nodes in the network. Let $G[A] = (V, E_A)$ denote the subgraph of G induced by A, i.e. (u, v) is an edge of $G[A]$ if there is a possibility that u delivers a packet to v under A. For instance, if A is a naive broadcast strategy where each node delivers a packet it receives to all nodes it meets within a time interval T, then (u, v) is an edge of $G[A]$ if the probability that u meets v within T is positive. We will refer to $G[A]$ as the *delivery subgraph* induced by A. Note that $G[A]$ along with the probabilities $p_e, e \in E_A$ define a probability space of random graphs (the Erdős-Rényi model $\mathcal{G}(n, \{p_{u,v}\})$ [12]). The probability that A successfully delivers a packet from s to t is the probability that there is a (directed) path from s to t in a random graph H chosen from this space. This probability is often denoted by $Conn_2(G[A])$ (or $Rel_2(G[A])$ for undirected graphs) in the network reliability literature [13]. The notation implicitly assumes the source s and the destination t to be known in advance. Also in this literature, $G[A]$ is called a *probabilistic graph*, so is G for that matter.

Based on the above discussion, our objective is thus to maximize $Conn_2(G[A])$ with a reasonable overhead. The maximum number of packets that A could produce is precisely the number of edges of $G[A]$. Thus, a very natural constraint to our optimization problem is to put a limit, k, on the number of edges of $G[A]$.

To summarize, the centralized version of our problem can be formulated as follows. We are given a probabilistic graph G, i.e. a graph along with a probability function $p : E(G) \to [0, 1]$, where p_e represents the probability that a packet can be delivered along edge e at a random point in time. The problem is to choose a delivery subgraph $G[A]$ of G with at most k edges, such that the probability $Conn_2(G[A])$ that there is a path from s to t in a graph H chosen randomly from the probability space defined by $G[A]$ is maximized. To the best of our knowledge, this problem and its complexity has not been addressed in the literature. Also, there is the corresponding distributed version of the problem where the delivery subgraph $G[A]$ is to be constructed distributively, yielding a routing protocol.

In this paper, we will present a practical solution to this problem in the context of ICMAN, involving the following steps: (a) proving the hardness of our optimization problem and the hardness of computing the objective function $Conn_2(G[A])$ (Note that the hardness of $Conn_2(G[A])$ is well-known, but that does not imply the hardness of the optimization problem); (b) giving an algorithm to approximate $Conn_2(G[A])$; (c) devising a mobility model to estimate the contact probabilities p_e; and (d) designing a routing protocol for the problem based on the contact probabilities computed in step (c) and insights obtained from step (a), and showing the protocol's effectiveness by comparing it with other protocols for probabilistic routing.

Remark: if we also consider using Erasure Codes [14] for dada transmission, we can add an additional constraint in terms of the code rate, keeping the objective function the same. Some previous work on routing in DTN has considered this dimension [15], [16].

III. Complexity of the Routing Problem

Computing the connectedness probability in a random graph is very hard (even for graphs with bounded degree like in our case). There is a vast literature on this problem. In the probabilistic sense, see [12], [17], for example. In the computational complexity sense, the problem is #P-Hard, as it is precisely the well-known reliability problem for two terminals [13]. A minor point: when G is directed, the problem is often referred to as the *s,t*-connectedness problem denoted by $Conn_2(G)$. In the classic paper [18] Valiant was the first to establish that both $Rel_2(G)$ and $Conn_2(G)$ are #P-complete [19].

The optimization problem, however, may not be hard even though computing the objective function is hard. This point is a little bit subtle. Given an integer a, computing the function $f(a) = 2^a!$ takes exponential time; yet, the optimization problem of finding which member a of a set A of integers has the largest $f(a)$ has the same complexity as sorting.

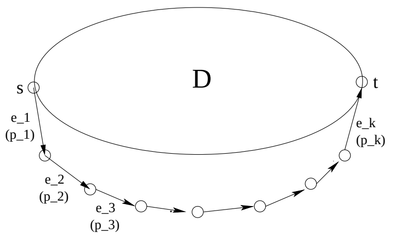


Fig. 1. Construction of G'' for proving the #P-hardness of maximizing *s*,*t*-connectedness

To this end, we consider the optimization problem of finding a delivery subgraph $D = G[A]$ with at most k edges which maximizes $Conn_2(D)$. We will show that[1]

Theorem 1: The routing problem on random graph is #P-hard.

Proof: We will reduce the 2-terminal connectivity problem ($Conn_2$) itself to our problem. Consider a generic instance of $Conn_2$ where we are given a directed graph $D = (V, E)$ along with a source s, a destination t, and all the edge probabilities p_{ij}. The problem is to compute the probability $Conn_2(D)$ that there is a path form s to t in a random graph chosen from the probability space defined by D. Note that all p_{ij} are rational numbers represented by a numerator and a denominator which are integers. Let c be the least common multiple of all the denominators, then the number of bits to represent c (i.e. $\log_2 c$) is certainly polynomial in the input size.

Consequently, if we have a procedure to decide if $Conn_2(D) \le c'/c$ for any integer $c' \le c$, then we can compute $Conn_2(D)$ by a simple binary search. We will prove that an algorithm solving our optimization problem can be used to decide if $Conn_2(D) \le c'/c$.

Construct a graph G as shown in figure 1, where the upper part is D itself, and the lower part is a simple path from s to t consisting of exactly $k = |E(D)|$ edges. The edge probabilities of the upper part is the same as those of D. The edge probabilities of the edges $e_1, \dots, e_k$ of the lower part is chosen so that $p_1 \cdot p_k = c'/c + \epsilon$, where $\epsilon < 1/c$. Our optimization problem is to compute a subgraph H of G with $|E(H)| \le k$ so that $Conn_2(H)$ is maximized.

Let A be any algorithm solving our problem. It is easy to see that A will either return the upper part or the lower part of G. If A does return the lower part, then $Conn_2(D) > c'/c$; otherwise, $Conn_2(D) \le c'/c + \epsilon$. But, by the way c was chosen, $Conn_2(D)$ is exactly a multiple of $1/c$; hence, A returning the upper part implies that $Conn_2(D) \le c'/c$. Consequently, A can be used to decide if $Conn_2(D) \le c'/c$, implying that our problem is at least as hard as $Conn_2$ by the analysis above. ■

[1]We thank Prof. Charles Colbourn for fruitful discussions leading to this proof.

Given this negative result, one can envision two general approaches:

1) Find a polynomial-time computable function $p(G[A])$ which approximates $Conn_2(G[A])$ well. Then, devise an algorithm A that maximizes $p(G[A])$. Note that, $p(G[A])$ can also be used to compare the outputs of different routing algorithms; thus, it is useful in its own right whether or not we can devise an algorithm optimizing $p(G[A])$.
2) Find a routing strategy (heuristic) A for which $Conn_2(G[A])$ can be reasonably computed or estimated.

In the following sections, we present our results on both approaches.

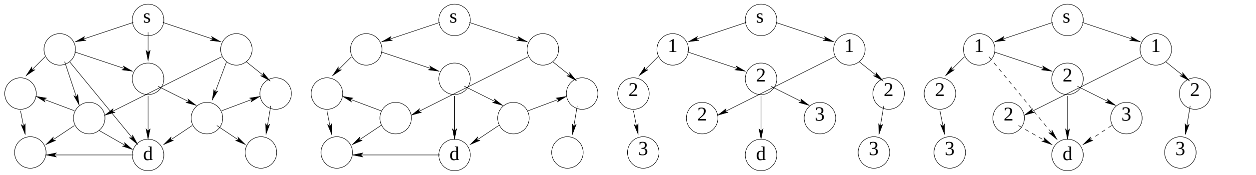
IV. Approximation Algorithm for Computation of Delivery Probability

In light of the discussion in the previous section, we propose an approximation algorithm for computing the delivery probability from source s to destination d in a network that is modeled as mentioned before: a directed graph $G = (V, E)$, where edge e exists between two nodes u and v with probability $p_e(u, v) =$ *contact probability* of u and v, as shown in Figure 2(a). First, we construct another graph $G_k = (V, E_k)$ from the graph G by having each node (starting from s onwards) choose at most k edges to downstream neighbors, and deleting all other edges not chosen, as shown in Figure 2(b). Second, we modify the weight of each edge in G_k to be $w_e = -1 * \log(p_e(u, v))$ for all nodes u and v, and call this new graph as G'_k. Third, we construct a shortest path tree $G_{sp} = (V, E_{sp})$ from G'_k as shown in Figure 2(c), and assign a level number to each node in a breadth first manner. Fourth, we replace the weight of each edge w_e in G_{sp} with $p_e(u, v)$, as in the original graph G. Finally, we add *special edges* (dotted edges in Figure 2(d)) between any node v and destination d in graph G_{sp} that were connected by an edge $e \in E$ in the original graph G, to get our *delivery subgraph* $G' = (V, E')$.

Let $P^d(u, v)$ denote the delivery probability of node u to node v. We apply our Algorithm 1 to this graph G' starting with any node $u \ne d$ with maximum assigned level number, to obtain the delivery probability $P^d(s, d)$ of the source s to the destination d. For each chosen node u, we consider all outgoing edges from u to nodes $v_1, v_2, ...v_k$ say, and get a list of probabilities $p_1, p_2, ...p_k$, where $p_i = w_e(u, v_i) * P^d(v_i, d)$. Then, we can compute the delivery probability from u to d as

$$P^d(u, d) = 1 - \Pi_1^k (1 - p_i)$$

This process is repeated with decreasing level numbers till node s is reached, and the required probability $P^d(s, d)$ is computed.



(a) $G = (V, E)$ (b) $G_k = (V, E_k), k = 2$ (c) $G_{sp} = (V, E_{sp})$ (d) $G' = (V, E')$ where dotted edges are *special edges*

Fig. 2. Steps in preparing a network graph for the application of Approximation Algorithm 1

Algorithm 1 : Approximation of Delivery Probability

Input $\leftarrow G = (V, E), s, d$
$P^d(d, d) \leftarrow 1$
$L \leftarrow$ maximum assigned level number
while $L \geq 1$ **do**
  for all $u \in V, u \neq d$ with assigned level number L **do**
    $i \leftarrow 1$
    for all outgoing edge $e \in E$ from u **do**
      $v \leftarrow$ head of edge e
      $p \leftarrow$ weight of edge e * $P^d(v, d)$
      $P[i] \leftarrow p$
      $i \leftarrow i + 1$
    end for
    $p1 \leftarrow 1$
    for $j \leftarrow 1$ to $(i - 1)$ **do**
      $p2 \leftarrow 1 - P[j]$
      $p1 \leftarrow p1 * p2$
    end for
    $P^d(u, d) \leftarrow 1 - p1$
    if $u = s$ **then**
      print $P^d(s, d)$
      exit
    end if
  end for
  $L \leftarrow L - 1$
end while

Algorithm 2 : Optimal computation of Delivery Probability

Input $\leftarrow$ All paths $PATH_1, PATH_2, ...$ from s to d
$m \leftarrow$ total number of paths
$P^d(s, d) \leftarrow 0$
for $n \leftarrow 1$ to m **do**
  coefficient $\leftarrow (-1)^{n-1}$
  for start $\leftarrow 1$ to m **do**
    All edges are un-marked
    for path-index $\leftarrow$ start to (start$+n-1$) modulo n **do**
      Mark all edges in path $PATH_{path-index}$
    end for
    term $\leftarrow$ product of all probabilities of marked edges
    $P^d(s, d) \leftarrow P^d(s, d) +$ (coefficient*term)
    if $n = m$ **then**
      break
    end if
  end for
end for
print $P^d(s, d)$

The optimal approach for computing the delivery probability from a source s to a destination d would include the following steps:

1) Calculate all possible paths from s to d
2) Apply Algorithm 2 to compute the delivery probability by rules of inclusion and exclusion

We simulated using Matlab [20] a small directed graph with 25 nodes with a given distribution of pair-wise contact probabilities $p_e(u, v)$ to evaluate the performance of our suggested approximation algorithm in comparison to the optimal algorithm. We chose 12 distinct source-destination {*s*,*d*} pairs and only computed the delivery probabilities $P^d(s, d)$ through the two algorithms, without sending any actual traffic. Figure 3(a) shows the results of our simulation runs where our approximation algorithm is seen to perform within 88% of the optimal algorithm on an average. For each {*s*,*d*} pair, we further simulated for 20 different pair-wise contact probability matrices. Figure 3(b) shows the relative performance of the approximation algorithm with respect to the optimal algorithm for each {*s*,*d*} pair, averaged over all the runs with different contact probability matrices for that {*s*,*d*} pair. Once again we find our approximation algorithm to perform within 85% to 90% of the optimal solution.

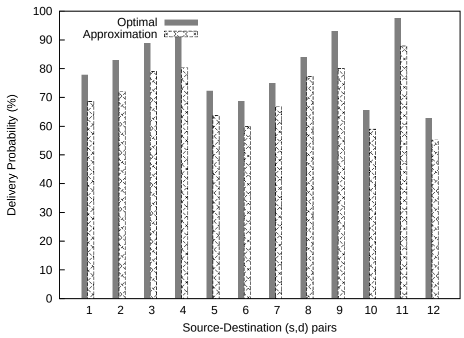
V. A Model for Computing Contact Probability

In this section, we present a model for computing contact probability of network nodes in the context of intermittently connected mobile ad hoc networks. The model is based on experimental data we gathered from real-world user movements' traces. Then, in the next section, the effectiveness of this model is illustrated by its usage in our routing algorithm.

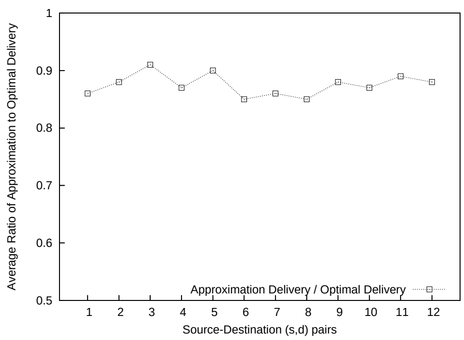
Consider a node X whose set of hubs is $\mathcal{S}$. From our earlier work in [9] we have verified that X's staying time at a hub $h \in \mathcal{S}$ roughly follow a power law distribution with exponent λ_h^X. After staying at h, X moves to another hub $h' \in \mathcal{S}$ with hub transitional probability $\beta_{hh'}^X > 0$. Obviously,

$$\sum_{h' \neq h} \beta_{hh'}^X = 1, \quad \forall h \in \mathcal{S}.$$

In the data set we analyzed in [9], we were unable to gather enough information about the inter-hub transition time. Hence, in our model the time it takes for X to move from h to h' is assumed to be exponentially distributed with parameter $\lambda_{hh'}^X$.



(a) Absolute performances with fixed contact probability matrix



(b) Relative performance averaged over different contact probabilities

Fig. 3. Performance comparison of our approximation algorithm for delivery probability with the optimal solution

This assumption can be relaxed/changed when there is better experimental data on users' mobility. The model proposed in [21] can be used, for instance. The following analysis can easily be modify to adopt the new and supposedly more correct distribution of hub transition times.

The movement pattern can be modeled with a Semi-Markov Chain (SMC) [22] whose embedded Markov chain (EMC) has state space

$$I_X = \mathcal{S} \cup \{(h, h') \mid h, h' \in \mathcal{S}, h \neq h'\}.$$

Here, the states (h, h') represent X being on the move from hub h to hub h'. The sojourn times at the "hub states" h are power-law distributed and the sojourn times at the "hub transition states" (h, h') are exponentially distributed. The transitional probability p_{ij}^X of the corresponding EMC can then be computed as

$$p_{ij}^X = \begin{cases} \beta_{hh'}^X & i = h \text{ and } j = (h, h') \\ 1 & i = (h, h') \text{ and } j = h' \\ 0 & \text{otherwise,} \end{cases}$$

where $i, j \in I_X$ and $h, h' \in \mathcal{S}, h \neq h'$. The states in this SMC when node X moves from hub h to hub h' are illustrated in Figure 4.

Suppose we have all these SMCs modeling the movements of nodes within their respective hub lists. Consider two nodes X and Y whose hub sets are $\mathcal{S}$ and $\mathcal{T}$ respectively. Define $\mathcal{R} = \mathcal{S} \cap \mathcal{T} \neq \emptyset$. We would like to calculate the probability that X meets Y at some particular time t in the future with t sufficiently large (i.e., at equilibrium), and also the probability that X meets Y at a particular hub $h \in \mathcal{R}$ at time t.

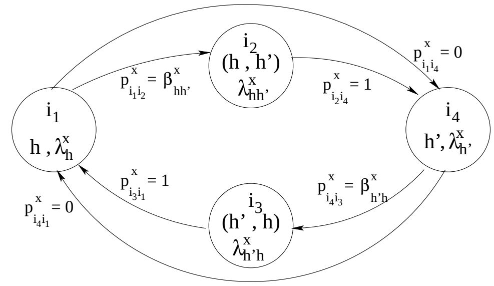


Fig. 4. States of Markov Chain for movements between hubs h and h'

Let I_X and I_Y be the state spaces of the SMCs capturing the movements of X and Y, respectively. In order to track the relative positions of X and Y together, define a SMC $\{Z_t \mid t \geq 0\}$ which is the Cartesian product of the SMCs for X and Y. For any $x \in I_X, y \in I_Y$, the sojourn time of the state (x, y) of Z has the distribution of the random variable which is the minimum of the sojourn times of x and y. Since the sojourn times at x and y are either exponential or power-law with known parameters, it is easy to compute the distribution of the sojourn time at (x, y). We omit the detailed formulae, which come from relatively simple exercises.

The corresponding EMC of (Z_t) has state space $I = I_X \times I_Y$. To characterize this process, we need to compute the jumping probabilities from a state (i_1, i_1') to another state (i_2, i_2'), where either $i_1 = i_2$ or $i_1' = i_2'$. (With probability zero the two chains for X and Y jumps at the same time.) Consider any state (i, i'). If the sojourn time T_i at state i of X is smaller than the sojourn time $T_{i'}$ at state i' of Y, then with probability p_{ij}^X the chain moves to state (j, i'), for some $j \in I_X$. Conversely, if $T_i > T_{i'}$, then the chain moves to state (i, j'), for some $j' \in I_Y$. Consequently, we can compute the jumping probabilities of the EMC for (Z_t) by conditioning on the event $\{T_i > T_{i'}\}$ and its complement. Again, we omit the detail case-by-case formulae.

The states of X and Y can be assumed to be overlapping. (If the states are not overlapping, X and Y will never meet.) The EMC of the Cartesian-product SMC as defined above is ergodic as long as the EMC for X and Y are ergodic (under most normal circumstances, otherwise we can disturb the chain by adding a few transitions with infinitesimal probabilities). In this case, we can easily compute the occupancy probabilities at equilibrium of any state (i, j) of the product chain by solving for the stationary distributions of the EMCs of X and Y. We are interested in only the occupancy probabilities $\pi^{XY}(h, h)$ of the states (h, h) where $h \in \mathcal{R}$. This is precisely the probability that X meets Y in h at equilibrium. Finally, the probability that X meets Y at equilibrium is the sum of $\pi_{h,h}^{XY}$ over all $h \in \mathcal{R}$.

Suppose X holds a packet it would like to transmit to a downstream neighbor towards a destination. It cannot hold the packet forever due to limited buffer size (and possibly delay requirements). Some routing strategy may require X to try its best to deliver the packet to (some of) the best neighbor(s) within a pre-defined time interval T. Consequently, given a time interval T and given that X is in some hub $h \in \mathcal{R}$, we are also interested in the probability that Y will be in h within T. Computing this probability is the same as computing the densities of the hitting times of the SMC corresponding to Y (probability that Y hits h given some initial distribution). There is no known general formula. Computationally however, there are methods to compute these densities using Laplace transforms [23] for larger chains or uniformization [24] for smaller chains.

VI. Edge Constrained SOLAR Heuristic

In this section, we present a Sociological Orbit aware Location Approximation and Routing (SOLAR) heuristic that makes use of mobility profile and hub transitional probability based computations for contact and delivery probability.

A. Edge-Constrained SOLAR-KSP

We chose our Static SOLAR KSP (S-SOLAR-KSP) algorithm proposed in [11] to form the base of this heuristic, with some additional modifications. In general, in this version of user-level routing protocol SOLAR-KSP, we assume that each user knows of every other user's mobility profiles and each user distributively does the following: First, every user computes the *contact probability* with every other user. In this work, we compute these probabilities based on the simulated mobility traces, as opposed to other various ways suggested in [3], [25]–[27] for example. Second, we represent the contact information between all users as a complete weighted graph $G = (V, E)$, where V is the set of all the users, and E is the set of weighted edges between every pair of users that have at least one hub in common. Let $P(u, v)$ be the contact probability of users u and v. Then the weight of edge (u, v) is given by $w(u, v) = \log(1/P(u, v))$. Now whenever a source user s has a packet to forward to destination user d, it applies the Algorithm 3 on the weighted graph to find a delivery subgraph to d that has at most $L(\leq |E|)$ edges. In other words, s iteratively uses Dijsktra's Shortest Path algorithm [28] to find the shortest path from s to d. In a single iteration, if the number of new edges encountered on the shortest path (that are not already in the delivery subgraph formed so far) is less than the remaining number of edges allowed under the edge constraint, then those edges are added to the delivery subgraph and the edge constraint parameter is adjusted accordingly. Also, to ensure that a new path is formed in the next iteration, the lowest weighted edge in the shortest path is deleted from the working graph. This algorithm terminates either if the working graph is exhausted, or if no more path exists between s and d, or if the edge constraint is met. Once this delivery subgraph is obtained, the source inserts this additional information into the header of all the packets and waits for all the next hop neighbors on all paths to appear. The intermediary users keep forwarding in accordance with this delivery subgraph till the destination is reached. We shall refer to this edge constrained SOLAR version as **EC-SOLAR-KSP**.

Algorithm 3 : Edge constrained delivery subgraph

Input ← Complete weighted graph $G = (V, E)$, Edge constraint L, source s, destination d
$DSG = (V', E')$ ← initial delivery subgraph with $V' \leftarrow V$ and $E' \leftarrow NULL$
while $|E|$ **do**
  apply Dijkstra's Shortest Path algorithm [28] to find shortest path from s to d i.e., $SP_{s,d}$
  if $SP_{s,d}$ == NULL **then**
    return DSG
  end if
  if $|\{e \mid e \in SP_{s,d},\ e \notin E'\}| \leq L$ **then**
    $L \leftarrow L - |\{e \mid e \in SP_{s,d},\ e \notin E'\}|$
    $E' \leftarrow E' \bigcup \{e \mid e \in SP_{s,d},\ e \notin E'\}$
  end if
  if $L == 0$ **then**
    return DSG
  end if
  $E \leftarrow E-$ the edge in $SP_{s,d}$ with least weight
end while
return DSG

B. Performance Comparison Results

We compare the performance of EC-SOLAR-KSP with that of a probabilistic routing (referred to here as **PROB-ROUTE**) based on [5] and Epidemic Routing (referred to as **EPIDEMIC**) [2]. In PROB-ROUTE, users each go through the "initialization" phase whenever they meet another user, where upon their contact probability is updated. When a pair of user do not meet for long, their corresponding contact probability is "aged". Also when a source has a packet to send to a destination it may calculate transitional probability through other users. The reader is referred to [5] for the detailed description and equations. In our implementation of PROB-ROUTE we used a value of 0.5 for all three parameters P_{init}, γ, and β, and allowed each user to forward a copy of a packet to at most 3 different neighbors with higher delivery predictability. In EPIDEMIC, users not only leverage upon the most instinctive broadcasting and buffering of data, but also rely upon user mobility to spread messages in the network (like the spreading of an epidemic). In order to limit the number of data transmissions in such a broadcast scenario, the so-called "summary vectors" were used to help users selectively exchange data packets. As for EC-SOLAR-KSP, we consider 3 variations: **EC-SOLAR-KSP1** with $L = |E|$, **EC-SOLAR-KSP2** with $L = 0.8 * |E|$, and **EC-SOLAR-KSP3** with $L = 0.6 * |E|$.

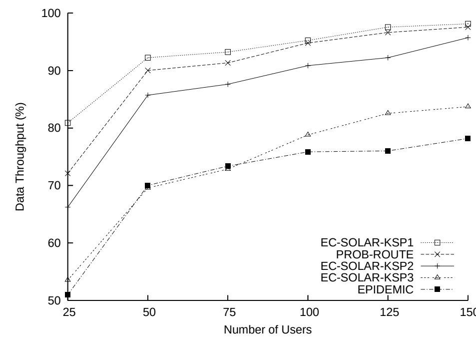
For simulation, we consider an ICMAN built within a campus consisting of several buildings (hubs) in accordance with the findings from our study [9] of an year long wireless users' mobility traces on ETH Zurich campus. In the Probabilistic Orbit model simulated, the users spend most of their time within a number of hubs, and intermittently move between hubs. To model realistic speeds of mobile users within such a network, we consider the work in [29], [30] and fix the Inter-Hub and Intra-Hub time/speed parameters, along with the

TABLE I
SIMULATION PARAMETERS

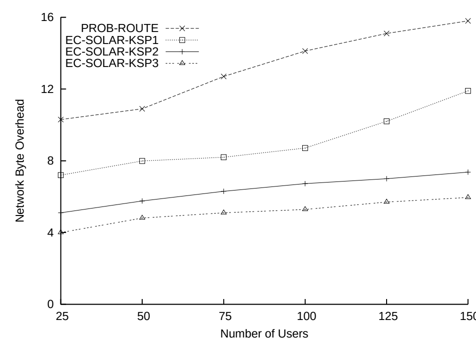
GENERAL PARAMETERS			
Simulation Duration (each run)	3000s	Terrain Size	1000m x 1000m
Number of Users (*Users*)	*Vary*, (Default= 100)	Radio Range	125m
Cache Size	*Vary*, (Default= 200 Packets)	Cache Timeout	*Vary*, (Default= 400s)
MAC Protocol	IEEE 802.11	Mobility Model	Probabilistic Orbit
ORBIT PARAMETERS			
Total Hubs	*Vary*, (Default= 15)	Hub Size	50m x 50m
Hub Stay Time	Power Law (k= $10^6/x^3$s)	Hub List Timeout	None
Hub List Size	Power Law (k= $0.7/x^2$, 2 to Total Hubs)	Inter-Hub Transition Time	Exponential (Mean= 40s)
Intra-Hub Pause	1s	Intra-Hub Speed	1m/s-10m/s
TRAFFIC PARAMETERS			
CBR connections	30 (120 packets each) Random	Data Payload	1460 bytes per packet

other simulation parameters and their default values or range of values as shown in Table I. In this study, due to space constraints we only present the protocol performances with a varying number of users as it is a significant factor in ICMAN settings, but we do study other variations as indicated in Table I. We chose 2 metrics to evaluate the performance of each protocol: *data throughput* = (data packets delivered)/(data packets generated); and *network byte overhead* = (total bytes transmitted)/(total data packets delivered);

As seen in Figure 5(a), EC-SOLAR-KSP1 performs the best closely followed by PROB-ROUTE. Since EC-SOLAR-KSP1 has the additional knowledge of mobility profile based network connectivity, it is able to compute the delivery probability via neighbors it has not met yet, unlike PROB-ROUTE where users have to meet at least once to update their initial contact probability. EC-SOLAR-KSP2 and EC-SOLAR-KSP3 have decreasing performances because of their increasing edge constraints. Epidemic performs the worst in the face of limited buffer. Also the fact that these results are observed within a limited time of $3000s$ takes its toll on EPIDEMIC, which is capable of "eventually" delivering all packets if time is not a constraint. Overall, all protocols do well with increasing number of users as it helps in finding a larger number of deliverable paths from source to destination. Figure 5(b) is not able to show the results for EPIDEMIC which has alarmingly large overhead. Amongst the rest, PROB-ROUTE incurs maximum overhead due to its arbitrary forwarding to neighbors just on the basis of larger delivery predictability. EC-SOLAR-KSP1 has lower overhead than PROB-ROUTE as it only forwards within the edge constrained delivery subgraph. The overhead in EC-SOLAR-KSP2 and EC-SOLAR-KSP3 is seen to decrease due to their increasing edge constraints. Overall, this "edge constraint" parameter gives us a good handle to a tradeoff between the desired throughput and the corresponding overhead.



(a) Data Throughput (%)



(b) Network Byte Overhead

Fig. 5. Protocol performance with varying number of users

VII. Related Work

None of the existing proactive, reactive or hybrid routing techniques proposed in literature for mobile ad hoc networks were deemed adequate in solving the routing problems within intermittently connected wireless networks. To that end, certain protocols adopt a "store and forward" kind of philosophy, wherein they hold on to data when a link is not available and transmit again to someone else at a future point in time. Authors in [2] proposed routing schemes using this philosophy. Similar work ([31], [32]) on data dissemination was also done for sensor and ad hoc networks. For delay tolerant networks, recent work in [4] assumed knowledge oracles and used a link capacity function to find minimum cost paths as a cascade of time varying links. Their usefulness is however limited by the practicality of their assumptions.

To gain more insight into the underlying user mobility, many researchers tried to model practical mobility in various ways to achieve different goals such as reproduce user movement in simulations [33], or use the mobility information in performing intelligent routing decisions [11]. Preliminary work on mobility modeling [34] was done mostly with Mobile Ad hoc NETworks (MANET) in mind. For example, some [35] used mobility pattern analysis to minimize radio link changes via appropriate selection of next hop within radio range. While the authors in [36], [37] performed physical location prediction via continuous short-term and short-range tracking of user movement, we had leveraged on our assumptions on "sociological orbits" to perform efficient routing within MANETs [38], [39] and ICMANs [10], [11].

Literature has also proposed several work on mobility trace analysis within campus-wide wireless networks. Authors in [7], [8], base their computational models on empirical mobility data that are filtered to provide stable mobility data sets spanning regular intervals of consecutive days. In most real systems however, we observe lots of irregularity in terms of wireless users' usage pattern of the network, where a user may not be present in the network consistently for the entire period of observation. To that end, our study of wireless users' sporadic mobility trace data [9] provided a technique for profiling users based on their socially influenced movement within wireless networks, which was shown to be beneficial to applications such as location approximation and routing. Compared to the most related (and yet much different) work in [40], our work primarily focussed on the "orbital" parameters, in particular on the *user-centric* parameters like the user mobility profiles and its applications, whereas [40] focussed more on AP-centric parameters.

In an effort to use the mobility information in routing decisions within temporally disconnected networks, literature suggests study on how mobility (controlled or not) affects routing protocols and network performance (e.g., network capacity) in various types of ad hoc networks including sensor networks with mobile sinks (or base stations), and delay tolerant networks [1], [41]–[43]. However, they did not deal with specific user mobility patterns. In [3], [25]–[27], the main focus was on the so-called "contact probability" of two users, which is oblivious to the specific locations (or "hubs") they visit.

The concept of Epidemic Routing was extended upon by the authors in [5], where they proposed a probabilistic routing scheme whereby each node maintains the so-called "delivery predictability" to each known destination, and uses this metric to make routing decisions. However, their delivery predictability may decay with time, unlike our contact probability information that remains valid for a longer time by virtue of the hub based mobility profile of nodes extracted from the underlying orbital mobility. In [44], the authors proposed a context-aware adaptive routing algorithm that takes into account the suitability of a node for carrying a message based on context information of the node at multiple dimensions. More recently, the authors in [6] suggested an algorithm that relies on vehicles to act as mobile routers, which connect disconnected sensor networks to a known destination.

In our work [11], we successfully used our mobility profiling techniques proposed in [9] to propose a suite of Sociological Orbit aware Location Approximation and Routing (SOLAR) strategies to deal with intermittently connected networks. In that work, although we used well known concepts of "contact probability" between pairs of users and "delivery probability" between a source and destination user, the way we compute these values is different. We base all our computations on the wireless user's mobility profiles and their "hub" transition probabilities in accordance with our sociological orbit framework [38]. This current work is the first to formally address the problem of routing in probabilistic graphs formed out of intermittently connected networks. This paper gives a model specifically for ICMAN, based on prior SOLAR works.

VIII. Conclusion

The mobility of wireless users with the added constraint of intermittent connectivity pose as one of the main challenges in devising effective routing protocols. In the literature, several protocols have been suggested based on the concepts outlined in [2], [3] which aim to infuse information within a network like an epidemic. Others [4]–[6] have studied network characteristics and proposed probabilistic routing techniques. Profiling wireless users based on their mobility has also been proven beneficial to routing. The suggested approaches to profiling [7], [8] have however varied, as well as the details of the mobility information required. In our previous work [9] we too have established a mobility profiling framework based on the sociological influences on wireless users' movements and further demonstrated its use in probabilistic routing within Intermittently Connected Mobile Ad hoc Networks (ICMAN) [10], [11].

In this work, we have analyzed our probabilistic routing framework mathematically and have provided some insight into the computational complexity of computing both the contact probability and the delivery probability, that is used by our previously proposed routing protocols [10], [11]. We have presented a rigorous formulation of the routing problem. We have studied and analyzed the hardness of computing an optimal delivery strategy. We have then proposed an algorithm to approximate the delivery probability of a delivery subgraph and presented its performance study in comparison with the optimal. We have also presented a mathematical model for analyzing mobility and computing the pairwise user contact probabilities. Finally, we have proposed an edge-constrained routing algorithm (EC-SOLAR-KSP) which makes use of such contact probabilities and have highlighted its superiority over other probabilistic and epidemic routing approaches proposed in literature.

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